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Universal conductance fluctuations in a multi-subband quantum well

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Abstract. We have investigated mesoscopic conductance fluctuations in a GaAs quantum well with five electrically quantized two-dimensional (2D) subbands occupied. In contrast to the case of a single occupied subband, where the fluctuations depend only on the component of magnetic field perpendicular to the plane of the electrons, in our multi-subband structures the conductance fluctuations occur even in a magnetic field parallel to the plane. We attribute the fluctuations to rapid electron–impurity scattering between different 2D subbands which leads to quasi-3D motion of electrons within the quantum well. We are able to confirm the 3D character of the fluctuations by correlations between the magnetoresistances observed when the magnetic field is applied at various angles relative to the plane of the quantum well.

In recent years there has been a great deal of interest in the transport properties of structures of reduced dimensionality, motivated both by the possibility of usable devices and by the availability of suitable technology for their construction. Usually the electrons are confined in one or more dimensions so that their motion is restricted to two (2D), one (1D) or even zero (0D) dimensions. There is also the possibility of more than one 2D subband being present in a system. Providing the number of subbands is not too great, the dimensionality of the system is not well defined and is in some sense intermediate between two and three. There is another sense in which conductors are often referred to as quasi-1D or 2D, which is occasionally confusing since it is different to the quantum mechanical confinement described above. In this case the relevant length scale is not the wavelength of the electron but the phase coherence length of the electron, l_ϕ which must be compared to the dimensions of the wire. For example, if l_ϕ is smaller than the length of a wire and its width, but larger than its thickness, the wire is said to be quasi-2D and when considering phenomena such as weak localization [1] or universal conductance fluctuations [2, 3] (UCF) it is necessary to apply formulae which are suitable for their dimensionality. Thus a wire can be quasi-2D even if its 'true' dimensionality is 3D, i.e. the electrons are free to move in all three directions.

In this paper we describe measurements on a wire which has five 2D subbands occupied and is quasi-2D or quasi-1D, depending on temperature, with respect to l_ϕ. We study UCF which are observed in the wire as the magnetic field, $\boldsymbol{B}$, is increased. The fluctuations are due to quantum mechanical interference of electron trajectories within the wire; the magnetic flux changes the phase difference between the trajectories [3]. For a strictly 2D system, only the component of $\boldsymbol{B}$ perpendicular to the motion of the electrons is important because only the flux enclosed by the electron trajectories contributes to the phase difference [4]. On the other hand, for a true 3D system such as a metallic wire, UCF [5] may be observed for all directions of magnetic field. For our wires, we are able to study the dependence of the UCF on the angle of $\boldsymbol{B}$ relative to the plane of the quantum well and hence study the dimensionality of the system. UCF are observable for all directions of $\boldsymbol{B}$ indicating that the motion of electrons in our multi-subband wire has a 3D character. We also report, for the first time, the angular correlation of UCF as $\boldsymbol{B}$ is rotated. When $\boldsymbol{B}$ is almost parallel to the length of the wire, we find that the UCF spectrum is sensitive to angle on a scale $< 0.1°$.

The samples were grown by molecular beam epitaxy and consist of a 40 nm GaAs/($Al_{0.3}Ga_{0.7}$)As quantum well doped by Si donors and Be acceptors in concentrations 6 and 4×10^{24} m^{-3}, respectively. The compensation was used to reduce the mobility and gain a larger amplitude of UCF. We have measured the electron concentration $n \approx 7.8 \times 10^{12}$ cm^{-2} which means that five 2D subbands are occupied in the 2DEG. The energy separation between the bottoms of adjacent subbands is $9, 16, 22$ and 38 meV in order from the lowest to the highest subband, assuming a square confinement potential. The mobility was found to be $\mu = 640$ cm^2 V^{-1} s^{-1}, corresponding to an elastic mean free path, $l = 15$ nm and a broadening of the energy levels of 13 meV. Electron beam lithography and dry etching techniques were employed to fabricate multi-terminal wires with a conducting width, w of about 0.5 μm and separation between adjacent pairs of voltage probes of 1 μm. The transport properties of the wires were measured using standard low-frequency lock-in techniques, magnetic fields up to 12 T and temperatures between 0.3 and 50 K. Note that $w \gg l$ and $\omega_c \tau < 1$ for all magnetic fields. The wires were mounted on a rotation probe which allowed us to monitor changes in the angle between the plane of the wire and the magnetic field $\boldsymbol{B}$ with accuracy better than 0.1°. The inset in figure 1 shows the geometry of rotation. Usually we rotated the wires so that the in-plane component of $\boldsymbol{B}$ was parallel to the current direction along the length of the wire. The angle θ was monitored by measurements of the Hall resistance R_{xy}, which is proportional to the perpendicular component of magnetic field, $B_\perp = B \sin\theta$. When $\boldsymbol{B}$ is parallel to the plane of the quantum well, $R_{xy}(B) = 0$. Measurements at high temperatures (50 K), when the UCF are negligibly small, allow us to find the parallel configuration with an experimental accuracy of 0.25°. To allow a simultaneous measurement of the angle and the UCF most of the experiments were carried out in the Hall configuration, R_{xy}, rather than the more usual longitudinal geometry, R_{xx}.

Figure 1 shows examples of conductance fluctuations for 0°, i.e. $\boldsymbol{B}$ parallel to the current, and 90° in the Hall geometry at $T = 2.5$ K. For $\theta = 90°$, the linear component of the Hall resistance R_{xy} has been subtracted leaving only the fluctuating part of the resistance. The fluctuations in perpendicular field are the well known UCF. We have analysed the detailed behaviour of the UCF in the perpendicular field for both R_{xy} and R_{xx} and found good qualitative and quantitative agreement with the standard theory [3]. The Lee–Stone (LS) correlation field ΔB_c, effectively the typical quasi-period of the UCF, and the amplitude of fluctuations both can be described by a single parameter, the phase coherence length of the electrons l_ϕ which is ~ 0.4 μm at 2.5 K. For brevity, we omit further details since UCF in GaAs quantum wires have been discussed in many previous publications (see e.g. [6, 7]). On the other hand, the conductance fluctuations (CF) with the magnetic field parallel to the 2DEG have not been discussed in the literature. In several earlier publications the scaling behaviour of UCF versus the angle has been investigated for a single subband 2DEG but no CF in parallel field were found [4]. Two major features of the CF at $\theta = 0°$ are readily seen in figure 1. First, the amplitude of CF is approximately the same for both field directions. Secondly, the quasi-period of fluctuations at $\theta = 0°$ is much larger than the period in perpendicular field.

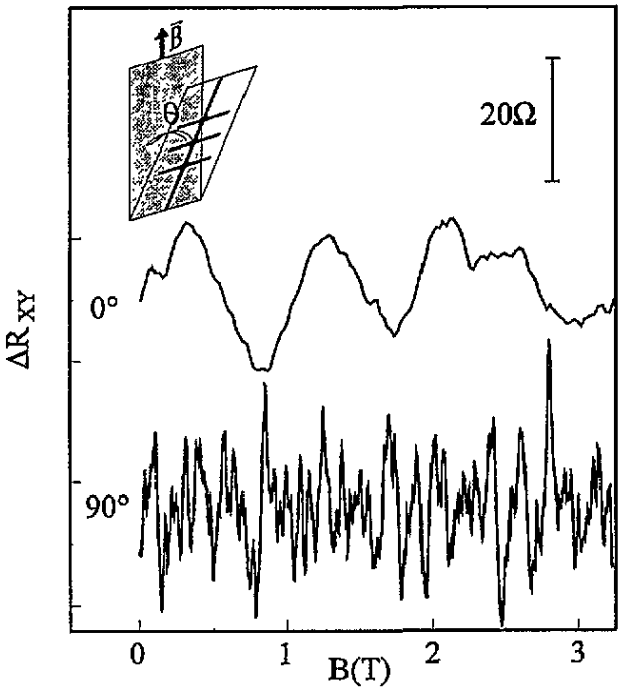

Figure 1. Conductance fluctuations in a Hall configuration for $T = 2.5$ K with the magnetic field in the plane of (upper curve) and perpendicular to (lower curve) the quantum well. The vertical scale is the same for both curves and the linear variation with magnetic field has been subtracted from the lower curve. The inset shows the definition of θ.

To show that the CF for 0° in figure 1 are not due to an accidental misalignment between magnetic field and the plane of the 2DEG, we plot in figure 2 a number of curves for a range of small angles around $\theta = 0°$. The quasi-period of the fluctuations is the same for all angles plotted, indicating that a small 'leakage' of the perpendicular component of $\boldsymbol{B}$ cannot be responsible for these CF. If this were the case, the quasi-period of the fluctuations would change significantly over the angular range shown and, in particular, would be proportional to $\sin\theta$. Furthermore, for small rotations, say $\pm 0.5°$, the first few fluctuations in low magnetic fields, $B < 1.5$ T, are the same on all the curves (see figure 2). This demonstrates directly that it is the parallel field component which is principally responsible for the fluctuations at small angles. On the other hand, at high magnetic fields the curves in figure 2 become decorrelated for angular shifts of less than 0.25°. This is attributed to the effect of the perpendicular component of $\boldsymbol{B}$ as discussed below.

We show first the difference between the curves in figure 2 is due to the perpendicular component of magnetic field, $\boldsymbol{B}_\perp$. This is most simply seen by noting that the magnetoresistance for, say, $\pm 0.5°$ is different to that at $-0.5°$ although the parallel field component is identical for the two cases. We can also present a more quantitative argument. With increasing total magnetic field, the difference in $\boldsymbol{B}_\perp$ between successive curves for different angles increases and at some point it reaches a value of $B_{\perp c} = C\phi_0/l_\phi^2$ where $\phi_0 = h/e$, is the flux quantum and C is a constant close to 1. At this point the difference in $B_\perp$ is sufficient to change interference between most of phase-coherent electron trajectories inside the sample (due to the Aharonov–Bohm effect), so the two curves are effectively

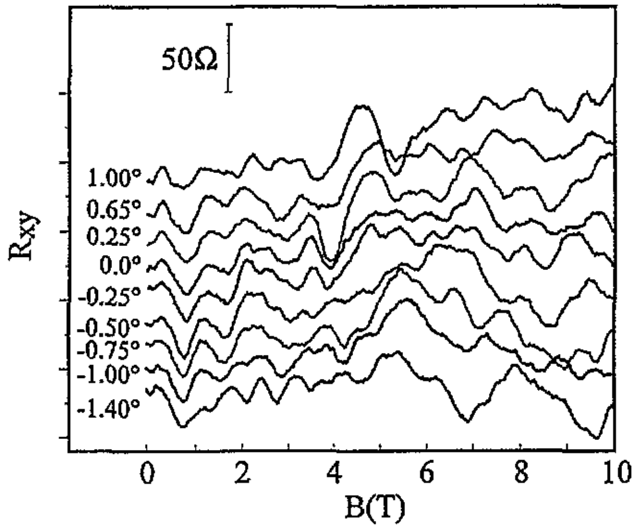

Figure 2. Hall resistance at $T = 2.5$ K for several directions of magnetic field close to the parallel orientation.

decorrelated. The difference in *parallel* field for small rotations is vanishingly small, ~ 1 mT at 10 T for a rotation of 1°. Unfortunately, our data do not allow us to provide a full statistical analysis of the correlation between resistance curves at various angles and in various magnetic fields; the task would require hundreds of traces at different angles. Nevertheless, for a simple quantitative analysis we can define an angular correlation field B_θ *as the lowest magnetic field where a magnetoresistance curve for a particular angle* θ exhibits a phase shift of 180° (i.e. a maximum becomes a minimum or vice versa) with respect to the curve at $\theta = 0°$. Figure 3 illustrates this definition of B_θ. Note that it does not matter if the curve labelled at $\theta = 0°$ has a slight angular error since $\sin\theta \approx \theta$ for these small angles. In figure 4 we plot the perpendicular component of the angular correlation field, $B_\theta \sin\theta$, for different small angles. The perpendicular correlation field is independent of the angle and is about 0.06 T. This value is in excellent agreement with the value (0.06 T) of the magnetic field which causes a similar change (i.e. from maximum to minimum or vice versa) of fluctuations on magnetoresistance curves for $\theta = 90°$. Note that the Lee–Stone correlation field [3], ΔB_c, is expected to be of approximately a third of this field [8] and is measured to be 0.02 T at this temperature for the perpendicular configuration.

We now address the behaviour of the conductance fluctuations due to the field parallel to the plane of the 2DEG. First we have calculated their root mean square amplitude (7 Ω at 2.5 K) and found that it is exactly the same value as the UCF amplitude in perpendicular field. This indicates strongly that CF for $\theta = 0°$ are also due to changes in electron interference. On the other hand, this does not necessary imply that the interference variation is due to the Aharonov–Bohm effect as in the case of perpendicular field. Other mechanisms may also be possible. For instance, scattering potentials of impurities in the sample may be modified by the strong parallel magnetic field. This would lead to CF of the same amplitude, even if only a few impurities are affected since the CF are sensitive to changes of a single scatterer in a phase-coherent area [9]. Furthermore, wavefunctions of conduction electrons or, semiclassically, electron trajectories are influenced by the parallel magnetic field which may also lead to CF [10, 11].

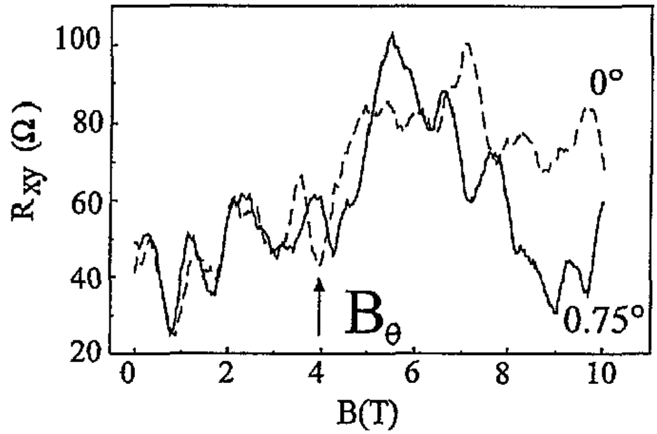

Figure 3. Magnetoresistance at angles of 0° and 0.75° showing the criterion for definition of $B_\theta = 4$ T.

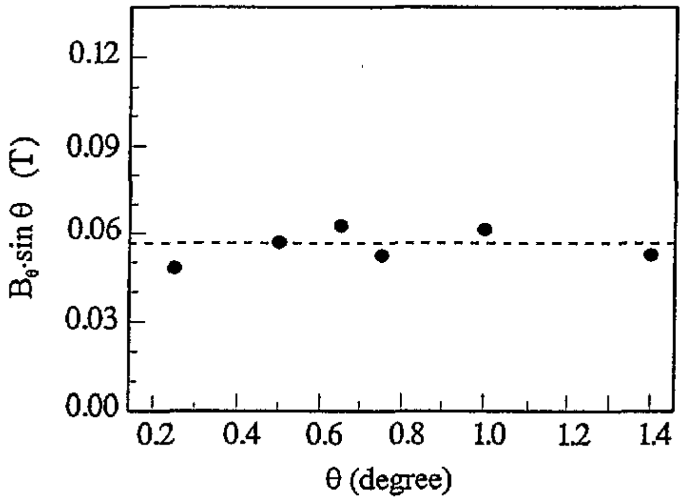

Figure 4. The perpendicular component of the angular correlation field, B_θ, plotted against angle of magnetic field. The dashed line is a guide to the eye.

The origin of the observed CF in parallel field becomes clear if we compare Lee–Stone correlation fields for the two field directions [3, 12]. The correlation field for the UCF in the perpendicular magnetic field $\Delta B_c = 0.02$ T while for the parallel field we found $\Delta B_c = 0.2$ T. If we assume that the CF in the parallel field are due to the Aharonov–Bohm effect, this order of magnitude difference can be interpreted as an order of magnitude smaller phase-coherent area S for the case $\theta = 0°$ since the correlation field $\Delta B_c \sim \phi_0/S$. For the case of a 3D metal film, $S = l_\phi^2$ for the perpendicular case and l_ϕ/d for the parallel case [3], where d is the thickness of the film, which gives l_ϕ/d for the ratio of the correlation fields. From this expression we find $d = 40$ nm, in agreement with the width of the quantum

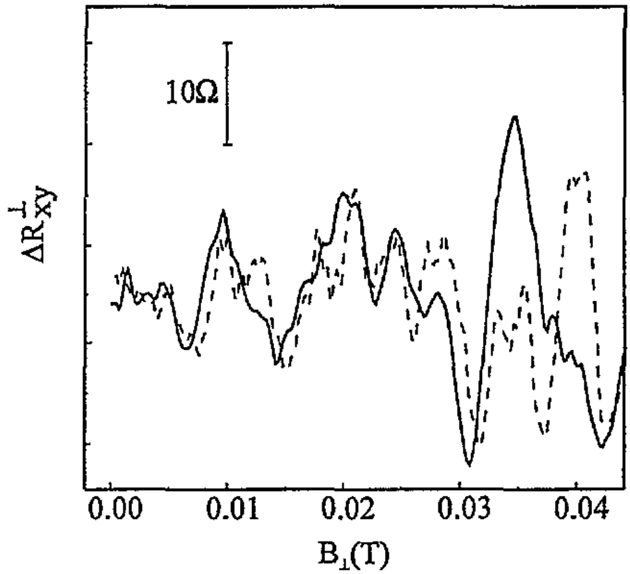

Figure 5. $\Delta R_{xy}^{\perp} = R_{xy}(\theta) - R_{xy}^{\parallel}(\theta = 0°)$ plotted against the perpendicular component of magnetic field for two values of the angle of magnetic field at $T = 2.5$ K. The solid line is $\theta = 0.50°$ and the dashed line for $\theta = 0.25°$.

well. Thus, we conclude that the observed fluctuations in the parallel field behave exactly as mesoscopic fluctuations expected for a 3D metal film of a thickness 40 nm.

Further confirmation that the CF for $\theta = 0°$ in our samples are due to the Aharonov–Bohm variation of the phase along quasi-3D trajectories comes from detailed consideration of the magnetoresistance curves in figure 2. If the effects of parallel and perpendicular fields *were* due to independent phenomena, then the magnetoresistance changes ΔR for any angle could be written as

$$\Delta R(B) = \Delta R^{\parallel} + \Delta R^{\perp} \tag{1}$$

where $\Delta R^{\parallel}$ and $\Delta R^{\perp}$ are fluctuating parts of the magnetoresistance due to parallel and perpendicular fields, respectively. Using equation (1) for the data shown in figure 2, we plot in figure 5 the fluctuating component of the Hall reistance $\Delta R_{xy}^{\perp}$ (that is, the measured Hall resistance *minus* the Hall resistance for $\theta = 0°$) against $B_{\perp}$ for two different small angles, 0.25° and 0.5°. If equation (1) were valid, the curves in figure 5 would be identical since $B_{\parallel}$ is essentially the same for both. In practice the plotted curves are very different. We conclude that equation (1) is false and the fluctuations due to the different components of magnetic field are not independent. Note that, although the Aharonov–Bohm effect itself *is* dependent only on the magnetic flux (a scalar quantity) threading a particular interference loop, figure 5 illustrates that it is *not* simply a matter of adding the effects of the flux in the two orthogonal directions to give the resistance. This absence of the scalar addition is entirely expected for UCF in the true 3D case and to explain this behaviour qualitatively, we plot in figure 6 a computer simulated trajectory for 3D diffusive electron motion in the limit $\omega_c\tau \ll 1$. Since a typical 3D trajectory is not flat, an electron moving along it picks up the phase from both parallel and perpendicular components of magnetic field. The resulting phase difference between any two trajectories is unique for any field orientation and, therefore, the effects of perpendicular and parallel fields cannot be separated.

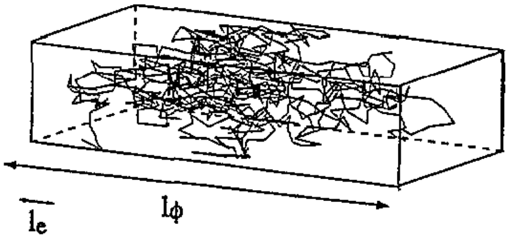

Figure 6. Simulation of classical 3D electron trajectories in the diffusive transport régime for $\omega_c\tau \ll 1$.

In conclusion, we have shown the mesoscopic fluctuations in the 'dirty' multi-subband 2DEG persist even when the magnetic field is parallel to the plane of the 2DEG. The fluctuations behave quantitative exactly as one would expect for UCF in the presence of 3D electron motion within the quantum well. At our highest magnetic fields, the magnetoresistance traces are de-correlated for angular rotations as small as 0.1° when the field is near parallel to the plane of the quantum well. We speculate that the reason we observe 3D behaviour when only a few 2D subbands are occupied is due to rapid intersubband scattering on a time scale much shorter than the phase-coherent scattering time. This is not unexpected since the elastic scattering lifetime is much shorter than that due to inelastic scattering and the large-angle impurity scattering should be very efficient at causing inter-subband transitions.

Acknowledgments

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